Modularity and Innovation

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Introduction

Managing a firm involves many decisions and parameters on the production process in a way that is typically very complex. The best way to structure the process is therefore often unknown, and firms somehow need to find ways to improve their production process. This process of production improvement is called innovation.

One principle to structure the production process is the principle of modularity. Modularity is the partitioning of the production process in semi-independent modules, each module being a subprocess of the larger overall process with separate functionality and production decisions. Modularity enhances clarity, reduces complexity, provides flexibility and has organizational advantages, allowing work in parallel and tasks to be solved independently (Miller and Elgård, 1998). However, modularity has a danger of miscoordination between modules. An innovation in one module may lead to conflicts and production problems in other modules, leading to significant delay in the innovation process (Ethiraj and Levinthal, 2004).

In this paper, we investigate the effect of modularity on the innovation process of the firm. We extend the work of Ethiraj and Levinthal (2004). They consider an innovation process in which the underlying optimal decision process has a modular structure itself. In that way, the optimal modular design of a firm is naturally to mimic the true modular design of the underlying optimal decision process. We, on the other hand, consider a model in which the underlying optimal decision structure has no modularity structure, but is represented by a non-modular network structure. We investigate if modularity is still beneficial to the innovation process in those cases.

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Basic Model

The innovation process is modeled by a group of firms, each firm trying to optimize a common performance function $h(x)$ (e.g. production). Firm performance depends on a vector x of $n = 30$ binary decision variables. Denote the set of decision as $V = \{1, \ldots, n\}$. Each decision variable x_v, $v \in V$, has its own decision performance function $f_v(x)$, and the performance function of the firm is simply the average of the performances of the individual decision variables:

$$h(x) = \frac{1}{|V|} \sum_{v \in V} f_v(x).$$

The performance function of one decision variable v is influenced by some of the other decision in V. This gives us an interaction network $G = (V, A)$, where the node-set V contains the nodes that represent decisions and the arc-set A contains an arc uv if decision u influences decision v. We assume that f_v at least depends on its own decision variable v. The value of $f_v(x)$ is drawn from an I.I.D. uniform distribution $U[0, 1]$, for each possible realization of the variables in $\{x_u : (u, v) \in A\}$, which are the decisions that influence decision v. This creates a highly irregular mapping with many local maxima, which is difficult to optimize globally.

The modular organizational structure of a firm is represented by a partitioning of V into m modules, $V_1, V_2, \ldots, V_m$. The decisions are assigned to the modules in order, such that if a firm has m modules, the number of decisions in module j is $|V_j| = \frac{n}{m}$, and decision variable i is in module j if $1 + \lfloor \frac{im-m}{n} \rfloor = j$ (i.e. when i is in the jth m-sized interval of $\{1, \ldots, n\}$). We only consider modularizations, such that $\frac{n}{m}$ is an integer (and thus each module contains the same number of decision variables). For example, if we have $m = 10$, we get $V_1 = \{1, 2, 3\}, V_2 = \{4, 5, 6\}, \ldots, V_{10} = \{28, 29, 30\}$. Given the decision performance functions, the module performance function of a certain module j, $g_j(x)$, is given by:

$$g_j(x) = \frac{1}{|V_j|} \sum_{v \in V_j} f_v(x).$$

A firm is able to evaluate the performance of each module. Note that although the value of $g_j(x)$ depends on the modular structure of the firm, the overall performance of the firm, $h(x)$, does not.

Evolutionary Algorithm

For the evolutionary algorithm, we consider a fixed population of $k = 10$ firms. Let us denote $z_j = h(x^j)$ as the fitness of firm j that chooses decision vector x^j. Now let us define z_{pop} as the average fitness of the entire population. This gives us $z_{pop} = \sum_{i=1}^{k} \frac{z_i}{k}$.

During an experiment, at each time step each firm will apply each of the processes that are active: local search, module recombination through firm selection, and module recombination through module selection.

Local search During the *local search* process, each firm does the following for each module $j \in \{1, \ldots, m\}$: Step 1) A random decision variable x_i is picked, such that variable i is in module j. Step 2) Variable x_i is flipped. This gives us a mutated vector x', which is equal to x, except that $x'_i = 1 - x_i$. Step 3) If the modular performance of j improves, that is, if $g_j(x') > g_j(x)$, x becomes x'. Otherwise, we keep x.

The modular structure affects the local search process in the following way. Each module experiments with a decision variable; hence, having more modules allows for more experimentation. However, note that a change of a decision variable x_v in module j is implemented, only if the performance of module j improves. Such a change may negatively affect other modules $j' \neq j$, though. Hence, unless the firm has only one "module", module performance and firm performance are not aligned, and local search in a modular structure may sometimes lead to lower firm performance.

Module recombination through firm selection During the module recombination through firm selection process, each firm i picks a single other firm i'. The probability that a certain firm i' is picked, is equal to $\frac{z_{i'}}{z_{pop}}$. Now a single module j is picked randomly from the modules of firm i and firm i' adopts the current decisions module V_j from firm i'.

This process let firms copy a module from other firms. The firm selection from which a module is copied is random, but better firms are chosen with higher probability. Since only one module is copied, having more modules slows down the recombination process.

Module recombination through module selection During the module recombination through module selection process, each firm i picks a single other firm i' with uniform probability. Also, a single module j is picked with uniform probability from the modules of firm i'. If $g_j(x^{i'}) > g_j(x^i)$ (i.e. this module currently has higher performance within firm i'), firm i adopts the values of the decision variables of module j in $x^{i'}$.

This process is similar to the firm selection process, except that this process only copies a module j from another firm i' if the module performance of j in i' is better. If firms consist of only one module, such that module and firm performance are aligned, then this process enforces firms always to imitate better-performing firms, and each step of the process would always lead to an improvement of the average performance of the population. However, if firms have a modular structure, then the copying process may sometimes decrease firm performance.

We compare 6 modular structures, $m \in \{1, 2, 5, 10, 15, 30\}$ and different underlying network structures G. For each network and modular structure, we simulate the process described above for 1000 periods, and we repeat the simulation 100 times.

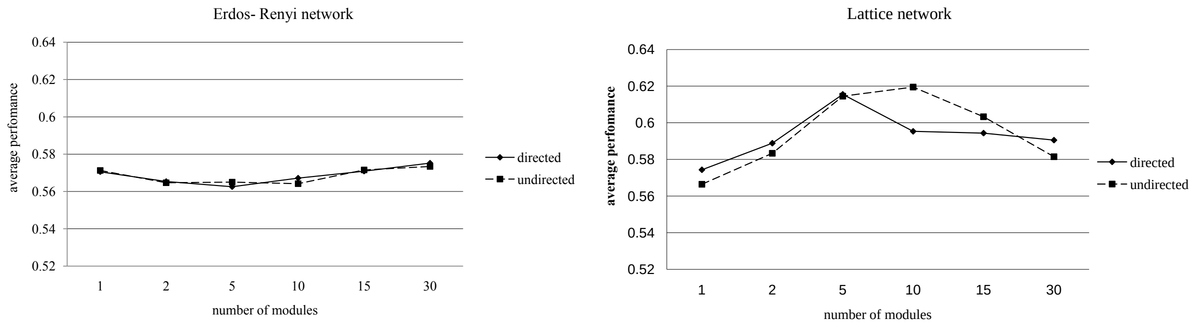
Simulation Results

We simulate the innovation process under different network structures of the underlying decision processes, G, and different modularity structures, m. All simulations converge, and we report the average population performance after convergence. At the end of the section we have a short word on the speed of convergence.

Regarding the network, we first consider Erdős-Rényi networks as described by Erdős and Rényi (1959). We draw 5 instances of a directed random networks (30 nodes, 120 links), and 5 instances of an undirected random network (30 nodes, 60 links). For each modularity level, we perform 100 simulations on each of the 5 networks and report the average of the 5×100 simulations. Figure 1a reports the results.

First, we observe that there is no significant difference between the directed and undirected networks. Averaging over the 6 levels of modularization, a two-sample Z-test on equal average performance of the directed and undirected random networks is not rejected ($Z = .21$, $p = .83$, $n = 3000$).

Second, we observe a nonlinear effect of the number of modules on the average performance. There is hardly any difference between having no modularization ($m = 1$) or having complete modularization ($m = 30$). A two-sample Z-test on equal performance is not rejected ($Z = 1.76$, $p = .079$, $n = 1000$). However, intermediate levels of modularization has a *negative* effect on the average performance. For example, the average performance



(a) (b)

Figure 1: Average population performance after 1000 periods for 6 different levels of modularity in which the underlying network structure of decision influence is: (a) a directed and undirected random network (Erdős and Rényi, 1959). The directed version randomly places 120 directed links across the 30 nodes, the undirected version 60 undirected links. 5 instances of the network were generated and for each random network 100 simulations were performed. Results report the population performance averaged over the 5×100 simulations. (b) a directed and undirected lattice network. Nodes are placed on a one-dimensional lattice (a circle). The directed version links up each node with its 4 neighbors on the right. The undirected version links up each node with 2 neighbors on the left and 2 neighbors on the right. Results report the population performance averaged over 100 simulations.

in case $m = 1$ is 0.571, whereas the average performance in case $m = 5$ is 0.564, which is a significant difference ($Z = -3.83$, $p < .001$, $n = 1000$).

We next consider a one-dimensional lattice network. The directed network links up each node with its 4 forward neighbors, whereas the undirected version links each node with its 4 closest neighbors, two on each side. The results of the simulations on the lattice network can be found in Figure 1b. First, we observe no significant difference between the performance on directed networks and undirected networks, just as in the case of random networks. Even though the figure suggests a difference for $m = 10$, a Z-test on equality of the performance on directed and undirected networks, taking the 6 modularity levels together, is not rejected ($Z = -.62$, $p = .54$, $n = 600$).

Contrary to the random network, in the case of a lattice network, we observe that intermediate modularity *improves* the performance of the innovation process. Having complete modularization ($m = 30$) is better than having no modularization ($m = 1$), ($Z = 3.56$, $p < .001$, $n = 200$). However, the best performance is reached when $m = 5$. In this case, taking the directed and undirected network together, the average performance is 0.615 in case $m = 5$, whereas it is 0.586 in case $m = 30$. This difference is significant ($Z = 6.09$, $p < .001$, $n = 200$).

Next, we consider small world networks as described by Watts and Strogatz (1998). These networks are generated by randomly rewiring links with probability p in an undirected lattice network. If $p = 0$, the network is a lattice network, and if $p = 1$, the network is a random network. Thus, the small world network model provides a continuous transition from a lattice to a random network. We consider one instance of a small world network for $p \in \{0, \frac{1}{40}, \frac{1}{20}, \frac{1}{10}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$, and perform 100 simulations on each network. Average performance is calculated, and the results of the experiments are reported in Figure 2a. The figure shows that the transition in the performance from lattice network to random network is more or less monotonic. The performance for intermediate levels of modularization is highest when $p = 0$ and gradually decreases for higher p.

In our simulations, we observe that the average population performance usually converges within 500 periods. Regarding the speed of convergence, in all simulations we observe that a higher modularity level m slows down the convergence process. Figure 2b shows the average performance at each step, in case of a small world network with $p = 0.1$, from which this effect can be seen. It appears that the advantage of experimenting with modules in parallel does not counter the disadvantage of modular conflicts.

Discussion

We observe a striking difference in the effect of modularization on the innovation process, when we compare random networks to lattice networks. Whereas intermediate modularization improves the performance of the innovation process in case of the lattice network, the opposite effect is the case for random networks.

In order to understand this result, it is important to understand how an arc uv between two decision variables $u, v \in V$ matters for the innovation process. Changing decision variable x_u may improve the performance of decision u, $f_u(x)$. However, it also affects the performance of all decisions v for which $uv \in A$, often negatively. Thus, a performance improvement of a decision u may prevent performance improvement of the firm if there are many links in G.

For the innovation process, however, it is the *module* performance that is more relevant. In the current model, modularity allows firms to experiment and evaluate the effect of their experiments at the level of a module. If the modules correspond to separate components in the influence graph G, then module performance and firm performance are aligned, and the innovation process would rapidly improve overall population performance. In general, however, an improvement in one module distorts the performance in other modules, in a similar way as described above. Hence, the crucial factor is the amount of influence one module has on the other modules, that is, whether the influence network between decisions, G, maps into a dense influence network between modules. We should expect the innovation process to work better in case the modules are quite independent.

It is with this respect that the random network and the lattice network starkly differ. Figures 3a and 3b show an undirected random and lattice network. Both networks have 60 (undirected) links of influence between decisions. Figures 3c and 3d show how these two networks map into links of influence between modules, when $m = 10$. We observe that the modules in the random network influence almost all other modules, whereas modules in the lattice network influence only two other modules. Thus, even though both the random and the lattice network have the same interaction density, intermediate modularization in a random network increases the interaction density, whereas it decreases it in case of a lattice network.

This fact explains why intermediate modularization is much more effective in case of a lattice network than for a random network. The crucial factor here is that the lattice network allows for such a low density interaction between modules, because the lattice network has a ordered structure itself that lines up well with the ordered structure of the modules.

Conclusion

In this paper we analyze the effect of modularization on the innovation within firms. We consider the same innovation process as Ethiraj and Levinthal (2004). However, whereas they assume that the underlying decisions are connected in a modular network structure themselves, we consider network structures that are not modular.

Even in those cases, we find that modularization can be effective for the innovation process. However, the effectiveness depends on the particular network structure considered. For lattice networks we find that modularization is beneficial, whereas this benefit gradually decreases when we move to a random network, until we reach a point where modularization actually hurts innovation. The crucial difference is that the ordered structure of a lattice networks allows for few interactions between modules, something that is not possible in case of random networks.

We showed that the interaction network has a big impact on the process of innovation. For future research, we consider the evolution of modularity itself as a promising direction. We could extend the evolutionary process in such a way that firms can reorganize their modularity structure. Then we can study how they adapt to the structure of the interaction network. Another direction would be to study the effect of perturbations of the performance function. This way, we can study the effectiveness of modularity in an competitive environment that changes over time.

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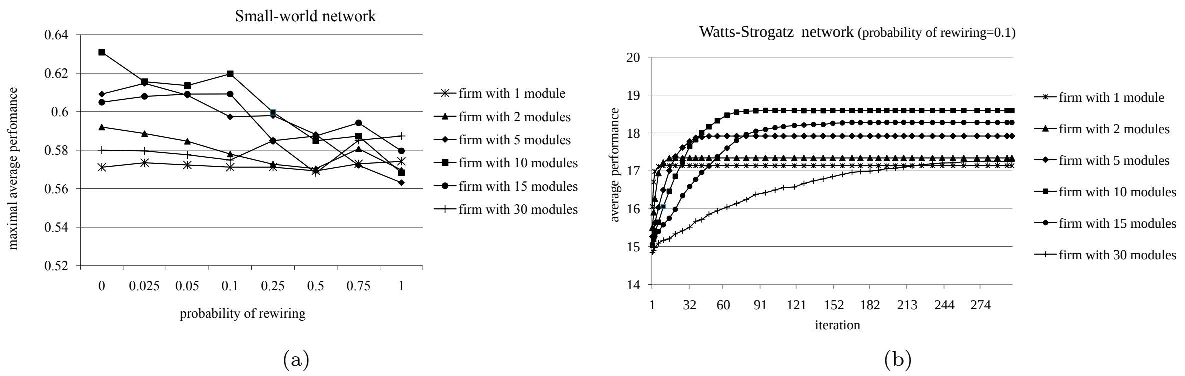

Figure 2: Average population performance for 6 different levels of modularity; (a) the underlying network structure of decision influence is a small world network (Watts and Strogatz, 1998) with probabilities of rewiring $p \in \{0, \frac{1}{40}, \frac{1}{20}, \frac{1}{10}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$. The underlying lattice network is as in Figure 1b. Results report the population performance after 1000 periods, averaged over the 100 simulations. (b) A time series plot for the average population performance at each step for a small world network with $p = 0.1$.

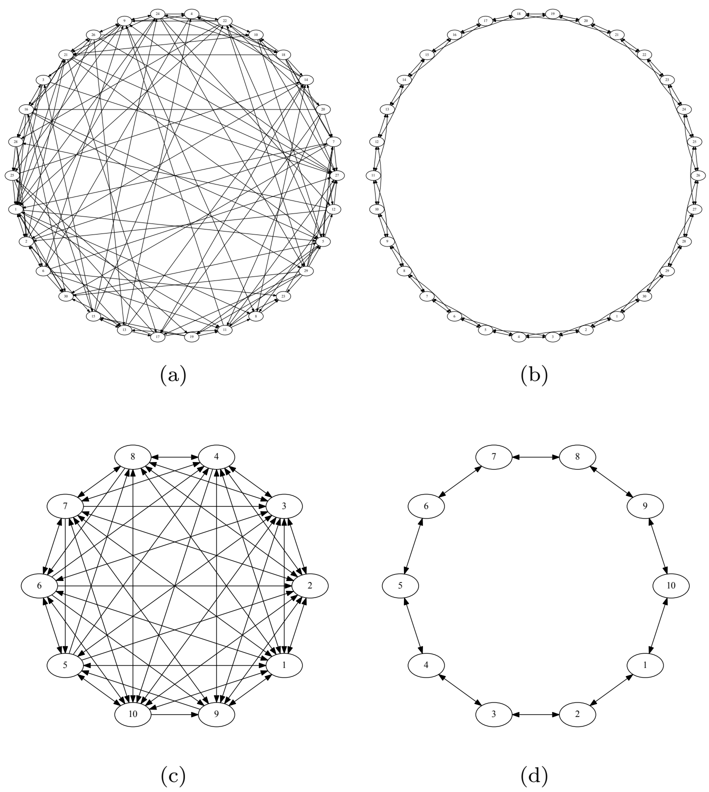

Figure 3: (a) instance of an undirected Erdős-Rényi network with 60 links; (a) an undirected lattice network, each node linked to its 2 closest neighbors; (c) the interaction structure between modules ($m = 10$) for the Erdős-Rényi network in (a); (d) similarly for the lattice network in (b).